Introduction:

In 1850 the Bach-Gesellschaft was formed with the purpose of publishing the complete works of Johann Sebastian Bach (1685-1750) as part of the centenary celebration of Bach’s death. The collected works, without editorial additions, became known as the Bach-Gesellschaft-Ausgabe. After the formation of the Bach-Gesellschaft, during 1873, Philip Spitta (1841-1894) published his biography of Bach. In this biography Spitta wrote that Bach used equal temperament. In other words, by the late 19th century, it was assumed by one of the more important writers on Bach that Bach used equal temperament.

During the 20th century, after the rediscovery of various kinds of historical temperaments, it became generally accepted that Bach did not use equal temperament. This theory was mainly based on the fact that Bach titled his collection of 24 preludes and fugues of 1722 as ‘Das wohltemperirte Clavier’, traditionally translated as the ‘Well-Tempered Clavier’. Based on the title, it was assumed during the 20th century that an unequal temperament was implied – equating the term ‘well-tempered’ with the notion of some form of unequal temperament.

But, are we sure that it was Bach’s intention to use an unequal temperament for his 24 preludes and fugues? The German word for ‘wohl-temperiert’ is synonymous with ‘gut-temperiert’ which in turn translates directly to ‘good tempered’. Does a ‘good temperament’ necessarily exclude equal temperament?

In 1954 the Bach-Gesellschaft commenced with the publication of an updated second edition of the complete works of Bach. This edition also included an additional section, namely the Bach-Dokumente, which contained references from all known books, letters etc. with information on J.S. Bach, including references regarding temperaments and tuning as they related to Bach as was then known at the publication date.

In this article we have collected every original quotation and reference on J.S. Bach’s tuning and temperament which has come down to us from the 18th century and added an 18th century quotation which connects Bach with equal temperament which is not included in the Bach-Dokumente, so as to answer the previous posed questions.

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2 Spitta, Johann Sebastian Bach, volume I page 651-653, relied on the fact that equal temperament was already known and described by Andreas Werckmeister (1645-1706) and Johann Georg Neidhardt (1685-1739) in the 17th century. Spitta further basised his opinion of Bach and equal temperament on the 18th century quotations by Marpurg, Versuch über die Musikalische Temperatur (1776), page 213 (see Bach Dokument 815 and appendix I); on Mattheson: Ehrenpforte, page 231 (appendix II); Mizler, Musikalische Bibliothek (1754) Band 4, Erster Teil, page 172 and 173 (appendix III); and Forkel, Über Johann Sebastian Bachs Leben, Kunst und Kunstwerke, page 17 (see Bach Dokument 666 and appendix IV).
3 http://www.qub.ac.uk/~tomita/bachbib/NBA-sup.html
5 With equal beating fifths tuned by ear which differs from the calculated equal temperament with in-proportion beating fifths as we shall explain later.
To understand the references and quotations properly and in context, it was necessary to comprehend what the ‘common knowledge’ was regarding temperaments during Bach’s lifetime. To this end, we re-read the important books from antiquity up to the 18th century that contained information on temperaments. These sources were read in their original language and within the original context.

To gain insight as to which temperaments were well known and in use during Bach’s lifetime, we first referred to two authoritative late 18th century music reference books, namely, Johann Nikolaus Forkel’s *Allgemeine Litteratur der Musik* first published in Leipzig during 1792 and Johann Siegmund Gruber’s (1763-1835) *Litteratur der Musik*, also first published in 1792.

In both these books, Forkel and Gruber, included titles and with synopsis of all publications on music then known - i.e. as at the end of the 18th century. Both these compendiums specifically contained chapters on temperaments and tunings and both authors mentioned writers on temperament from antiquity up until the end of the 18th century. Using these 2 sources as starting point, we worked our way back through the sources listed from the 18th century to antiquity.

For the temperaments compiled, we included information regarding the difference between practical and theoretical temperaments, but did not include theoretical temperaments prior to Bach’s birth if they did not provide useful information to assist in understanding the Bach preferences and quotations. For the same reason we did not include tuning systems that were only documented after Bach’s death.

The article is in 2 parts. The first part starts with a short historical overview of the development of temperaments from antiquity to the mid-18th century as follows:

1. Classical Antiquity
2. The Middle Ages to 16th Century
3. 17th century Netherlands
4. 17th century France
5. Early 17th century Germany
6. 17th and 18th century England
7. Late 17th century and 18th century Germany
8. The introduction of the equal temperament (*gleich-schwebende Temperatur*) in Germany in the 18th century.

The first part ends with a synopsis of part 1.

The 2nd part of the article gives the translations of the Bach Documents with the additional quotation added that establishes a link between Bach and equal temperament.

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6 We did not use quotations from second or third generation literature, as these may already have been interpreted out of the context. We also did not want to confuse the issue under investigation with all kinds of different modern interpretations on temperaments.

7 For readers who are not able to read the original languages we provide translations into English.

8 For example, Vallotti’s (1697-1780) manuscript was written in the second half of the 18th century at a time when the important composers and performers already applied equal temperament. Vallotti’s temperament became well known after his manuscript was printed in the 20th century.
1. Classical antiquity

Almost all writers on music in the Baroque era mention two famous theorists from classical antiquity: **Pythagoras** and **Aristoxenus**. These theorists held opposite views regarding tuning. Hence, to several of these baroque musicologists, two different cults relating to tuning in music existed in the classical antiquity: Aristoxenus and his *Harmonicos* and Pythagoras and his *Canonicos*.

**Pythagoras**\(^9\) calculated all kinds of intervals in a variety of ways with the help of a monochord – a single-stringed instrument (to which an extra string was sometimes added) and a movable bridge that could be moved along the string/s to demonstrate various mathematical relationships between sounds. Pythagoras used this instrument to calculate and demonstrate different intervals.

![Monochord from J.A. Ban’s Zangh-bloemzel (1642)](image_url)

Placing the moveable bridge at 1/2 the length of the string produced an octave,
1/3 of the length of the string produced a fifth,
1/4 of the length of the string produced a fourth,
1/5 of the length of the string produced a major third,
1/6 of the length of the string produced a minor third.

Dividing the string of the monochord in different calculated ways was then used to produce all kinds of different intervals and so temperaments.

**Aristoxenus**,\(^11\) the counterpart of Pythagoras, considered that human hearing was a better and ultimate judge of intervals, as opposed to the Pythagorean method of calculating intervals, and so tuned by ear: the fourths a bit wider and the fifths slightly narrower than pure. His tuning method became known as the Aristoxenus way of tuning equal temperament.

**Antonius Gogava**\(^12\) translated Aristoxenus’ *Ἀρμονικῶν στοιχείω* (Latin: *Elementa harmonica*) in 1562\(^13\).

During Bach’s lifetime it was generally accepted that Aristoxenus was the first who described equal temperament by making all the fifths a bit narrow by ear\(^14\). After the translation of Aristoxenus *Elementa harmonica*, mainly two methods of temperaments became in use:

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9 For example, Mersenne: *Livre second des instruments a chordes* (1636) page 56, W.C. Printz: *Phrinidis Mitilenaei,Ander Theil* (1696) page 96 and Neidhardt *Sectio Canonis Harmnicici* (1724) in the introduction which do not have page numbers.

10 571 BC-495 BC

11 fl. 335 BC

12 1529-1569

13 The translation got criticized because of inaccuracies, but the book was often read in the 16\(^{th}\) century. A better translation was published in 1616 by the Dutchman Johannes Meursius (1579-1639).
1. **Meantone** (or modified meantone) by tuning the octaves pure, 11 fifths out of tune, the last one became the ‘wolf’ \(^{15}\) and most thirds pure \(^{16}\).

2. **Equal temperament** tuned by ear: octaves pure, fifths so little out of tune that they sound pure and the thirds as wide as the ear can accept/bear \(^{17}\).

At the end of the 17\(^{th}\) century, unequal circular systems without a wolf, such as Werckmeister’s temperaments, were described for players who wanted to play in more tonalities than was possible if meantone temperament was used.

### 2. The Middle Ages to 16\(^{th}\) Century

Before starting to describe theoretical and practical temperaments it is necessary to explain how theorists and musicians tuned prior to the invention of the modern-day electronic tuner.

A monochord \(^{18}\) may be divided into a great number of small intervals, mathematically, by means of a pair of compasses. Theoreticians would calculate all kinds of different tunings and the monochord would then be used to represent these calculated temperaments. Placing the movable bridge step by step on the calculated points was a rather time consuming method when tuning as one would need a third hand to move the bridge from note to note, pluck the monochord and play the note to be tuned on the instrument being tuned.

Musicians, on the other hand, did not need a monochord, as they generally tuned by ear. A well-trained ear is able to tune unisons, octaves, fourths, fifths and thirds with great precision. And fifths, which are out of tune one beat per second, can also be tuned quite easily \(^{19}\).

Back to the Middle Ages: If one tunes 11 fifths in the circle of fifths pure, the 12\(^{th}\) fifth is about a \(\frac{1}{4}\) of a semitone too large.

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\(^{15}\) A wolf is a fifth which is unacceptable out of tune.

\(^{16}\) Or almost pure

\(^{17}\) And from the beginning of the 17\(^{th}\) century onwards equal temperament was calculated as well.

\(^{18}\) Which was mostly 4 foot long (1 meter, 22 centimetre).

\(^{19}\) As is explained in detail in our 18\(^{th}\) century chapter.
There are many possibilities to complete/close the circle. During the Middle Ages a simple solution was used: eleven fifths (consecutively) were tuned pure and the last fifth was left impure – the last fifth being known as the ‘wolf’.

During the 16th century, mainly South European music theorists were concerned with different temperaments. All theorists realised that there was a problem which could not be solved, namely, that if one made the fifths pure, the result was that the thirds became out of tune and if one made the thirds pure, the fifths became out of tune. Several baroque musicologists tried to solve this problem in different ways.

One solution was the mean-tone system. 11 fifths being narrowed by $\frac{1}{4}$ comma and the last fifth left out of tune. The consequence of the choice of meantone temperament was that most of the thirds sounded pure\textsuperscript{20}, but only a limited number of tonalities and modulations were usable.

On the other hand, if one made the fifths 1/12 of a comma narrow, then the fifths would sound pure\textsuperscript{21} but the thirds are out of tune – this method of tuning being equal temperament. But this choice then allowed the possibility of an unlimited number of transpositions and modulations to be made in a piece of music.

In 1530 Giovanni Maria Lanfranco (1490-1545) described a temperament for keyboard instruments in his Scintille di musica. In the chapter: Del modo di accordar gli instrumenti (A method to tune instruments) he described how organs and other instruments were tuned. Lanfranco started with a set of general rules that needed to be followed when tuning instruments\textsuperscript{22}. In his general guidelines he stated that octaves should be 100% pure, intervals of a perfect fifth should be made smaller, intervals of a perfect fourth should be slightly bigger, and ‘ciascuna Terza maggiore’ which means: all major thirds should be as large as possible so that the ear could still tolerate it.

\textsuperscript{20} Or almost pure, depending on the kind of meantone system and its interpretation.
\textsuperscript{21} About one beat a second narrow.
\textsuperscript{22} Scintille di Musica (1533) pp. 132-134.
Hence an application of which was believed to be Aristoxenus’ method was: tune equal temperament by ear and not by theoretical calculations, making the fifths narrow and all the thirds too wide. Lanfranco showed: *How to tune keyboard instruments* and provided an accurate step by step method on how to tune. In short, he started with the F then tuned the following fifths tempered equally: F-C-G-D-A-E-B-F sharp-C sharp –G sharp.24 Here he stopped.25 Lanfranco then went back to the F where he started off from and tuned in tempered fifths downwards from F-B flat-E flat. And if one tuned every fifth the same (one beat) narrow, the last interval: ‘E flat – G sharp’ which is theoretically not a fifth, would automatically have been narrowed in the proper way.

Tuning in fifths from F upwards to G sharp and downwards to E flat can also be used for tuning meantone, ending with a ‘wolf’, but Lanfranco’s tuning ‘*all major thirds should be as large as possible so that the ear can still tolerate it*’ is different from the meantone systems with pure thirds or alternative meantone systems which contain thirds even narrower than pure.

In the context of Lanfranco’s time both meantone temperament and the Aristoxenus way of tuning equal temperament were well known. We will not jump to a conclusion as to which temperament Lanfranco intended for keyboard instruments, but rather note that Lanfranco’s temperament was different from the meantone systems in use at that time by making all the thirds as large as the ear can tolerate.

Gioseffo Zarlino (1517-1590) did not only deal with the meantone temperament in general, but also, specifically with the tuning of lutes and viola da gambas in his writings. With the

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23 On page 134-136
24 Using octaves to stay in the middle of the keyboard and using fourths and thirds to check out if the tuning is still okay. Lanfranco does not use the terms ‘F sharp or G sharp’, etc. but tunes ‘the black note between F and G, etc. The G sharp or A flat is ‘the black note between G and A etc.
25 And mentions that one fifths is not tuned.
help of a mesolabium\textsuperscript{26} (a mechanical method by which geometric means would be extracted graphically) it was possible to position the frets quite accurately so as to give an equal temperament. Zarlino was well-known in Germany during J.S. Bach’s lifetime. For example Mattheson quoted Zarlino several times in various books; Kuhnau mentioned him in the preface of his \textit{Biblischer Historien}; Fux quoted Zarlino in his \textit{Gradus ad Parnassum} (1725)\textsuperscript{27} and Neidhardt in his \textit{Sectio Canonis Harmonici} (1724).\textsuperscript{28}

\begin{center}
\textit{A geometric representation of equal temperament, from Zarlino’s Sopplimenti musicali (1558)}\textsuperscript{29}
\end{center}

Using (almost) equal temperament on lutes made it possible for composers in the 16\textsuperscript{th} and early 17\textsuperscript{th} centuries to compose music which modulated through all tonalities. Examples can be found in the compositions of Jacomo Gorzanis (mid-16\textsuperscript{th} century), Vincenzo Galilei (second half 16\textsuperscript{th} century) and John Wilson (first half 17\textsuperscript{th} century)\textsuperscript{30}.

Using the Aristoxenus way to tune a harpsichord in equal temperament by ear made it possible for John Bull to use enharmonic modulations in his hexachord fantasy from the \textit{Fitzwilliam Virginal Book}\textsuperscript{31} - which sounds completely out-of-tune in a mean-tone temperament.

\section*{3. 17\textsuperscript{th} Century Netherlands}

During the Dutch 17\textsuperscript{th} century the discovery of logarithms\textsuperscript{32} and root calculations paved the way for calculating equal temperament. The rise of tuning equal temperament that was based on calculations (and not tuning by ear) started in the early 17\textsuperscript{th} century in The Netherlands.

\textsuperscript{26} An invention of classical antiquity. See Zarlino's \textit{Le institutione harmoniche [1558]} chapter XIX and XX.
\textsuperscript{27} Written in Latin and translated by Mitzler in 1742, page 33.
\textsuperscript{28} Page 25.
\textsuperscript{29} Page 211
\textsuperscript{31} It could have been written for an archicembalo.
\textsuperscript{32} John Napier discovered logarithms: \textit{Mizifici logarithmorum canonis description} (1614). The original work was translated by Edward Wright in 1616.
Simon Stevin (1548-1620), a Flemish mathematician and military engineer, was one of the first to calculate the equal temperament and described the result in his *Van de Spiegheling der singkonst* (c.1605).

In 1659 the Dutchman Dirck Rembrandtz van Nierop (1610-1682) proceeded to perform further calculations on temperaments, quoted Simon Stevin in his *Wiskonstige Musyka* and published Stevin’s calculated equal temperament:\(^{33}\):

\[\text{In 1659} \]

In 1699 Claas Douwes (c.1650-c.1725) published his: *Grondig ondersoek van de toonen der musiek*\(^ {34}\). In his chapter ‘Proper tuning of keyboard instruments’, he explained how to tune using a practical method\(^ {35}\):

\[\text{‘dat men in het stemmen van klavierinstrumenten, alle de quinten, de bovenste klavier een weinig laager stemmet als recht suiver…want twaalf quinten zijn maar twee snipsels te klein, soo dat ider quint maar een sesde deel van een snipsel te klein moet zijn…ende soo is dan ook weg genomen het onderscheit van meerder en minder toonen; want soo de quinten alle gelijke groot gestemmet worden/ soo sijn de heele tonen ook alle gelijke groot.’} \]

Which means: ‘when tuning keyboard instruments, in tuning all the fifths, to make the highest note a little bit lower than of pure…since 12 fifths are only two ‘snipsels’ narrow, every fifth needs to be 1/6 ‘snipsel’ narrow…and doing this there is no difference anymore between bigger and smaller notes; if all the fifths are equally narrow/ the result will be that all the whole notes are equally big.’

4. 17\(^{th}\) Century France

Equal temperament was also known in France by the first half of the 17\(^{th}\) century.

In 1636, Marin Mersenne (1588-1648) published his comprehensive work: *Harmonie Universelle*. In this book he described and calculated rather accurately, all kinds of different temperaments, tuning systems from classical antiquity till even equal temperament with in proportional beating fifths. Mersenne’s *Harmonie Universelle* was well known in Europe, in fact, every important musicologist in the baroque mentioned Mersenne.

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\(^{33}\) It is a rather accurate calculation of equal temperament.

\(^{34}\) Properly investigations concerning musical notes.

\(^{35}\) Page 35
Unfortunately Mersenne’s Harmonie Universelle contained several misprints. He used wrong words such as ‘juste’ instead of ‘forte’ (pure instead of too high), there are missing parts in the texts, parts of the text which should have been removed, mistakes in his musical examples and even mistakes in page numbers. He apologises in his Nouvelles Observations for the mistakes, especially for the errors in the chapters on tuning the l’Epinette and on tuning the Organ.  

Some views on temperaments described by Mersenne.

‘If you want to tune the harpsichord with equal half-tones (equal temperament), you can start at any key.’

‘Mais je ne sçache personne que le sieur Gallé, qui ayt accommodé de cet accord (equal temperament) à l’Orgue & à l’Epinette...n’y ayant que les Tierces majeures trop fortes...qui blesse l’oreille de nos Practisiens, qui ne l’ont pas accostumée à cet accord.

‘But I only know mister Gallé who uses this temperament (equal temperament) for the organ and the harpsichord...because the thirds are too high...which insults the ears of our organists and harpsichordists, which are not accustomed to this tuning.’

‘Il est certain que l’Orgue & l’Epinette estans temperées selon le manche des Luths & des Violes, les concerts qui en réussiront, paraîtront plus justes, a raison de la convenance de leurs accords. Mais nos Practisiens ne sont pas d’avis de changer l’accord de l’Epinette,’

‘For sure, if the organ and the harpsichord are tuned accordingly to the necks of the lutes and the viols (in equal temperament as well), the concerts in which they participated should sound much pure, because of their tuning. But our organists and harpsichordists don’t follow the advice to change their tuning.’

‘Il semble que les Practisiens peuvent mieux accorder l’Epinette par demitons égaux, en faisant les Quites justes’.

‘It looks like it is better if the organists and harpsichordists tune their instruments with equal half-tones (equal temperament) by making their fifths pure.’

Mersenne and the fretted instrument players did not have issues with equal temperament, as we will see in Livre second des instruments a chordes, whereas the harpsichordists and organists in France during the 17th century did not like equal temperament.

Tuning the lute and theorbo

Mersenne explained in his Livre second des instruments a chordes what he meant by temperament. The distances between the frets on the neck of a lute being divided in half notes which are not equally wide, but in-proportion. The distances proportionally become shorter in

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36 Nouvelles Observations, p. 22 and 29.
38 Jean Gallé de Liège, he signed in 1626 a contract with the organ maker Andre Severin to teach the latter: ‘la façon de faire orgues, positives, regales, espinettes et clavis, lesquelles par son invention se pourront hauser et abaisser, s’accordantes à tous tons avec une harmonie meilleure qu’à l’ordinaire. See Hanquet: Documents oubliés concernant Jean Gallé (1959).
40 To be exact: one beat narrow.
such way that the octave being half of the length of the whole string: which result in equal temperament\footnote{Or a rather accurate approximation of equal temperament.}.

Before the invention of root and logarithmic calculations it was not possible to calculate the placement of the frets with mathematical accuracy. In the 16\textsuperscript{th} century (quote Mersenne):

‘Plusieurs Facteurs d’instruments divisent la longueur du Luth, ou de la chorde en 18 parties, don’t la 17 fait la premiere touche; & puis ils divisent le reste de la chorde en 18 parties, don’t ils en prennent encore 17 pour faire le second demiton, & ainsi consequemment jusques à ce qu’ils aient 8. ou 9. demi-tons\footnote{Livre second des instruments a chords, p. 48.},’

‘Afin que tous ceux qui touchent le Luth, scavents’ils usent de temperament d’Aristoxene.’

‘Several instrument makers divided the length of the lute, or the string, in 18 parts, and placed the first fret on the 17\textsuperscript{th} part. Then they divided the rest in 18 parts again and placed the second fret on the 17\textsuperscript{th}, to find the second half tone (note). And consequently went on till they had 8 or 9 half tones.’

‘At last everybody who touched the lute, knew that they used Aristoxenus temperament.’

Here Mersenne mentions Aristoxenus again who, as we have mentioned before, made all the fifths in the circle of fifths a bit narrow by ear, a practical method to close the circle of fifths which resulted in equal temperament.

Mersenne described the two cults in antiquity:

‘Les disciples d’Aristoxene ont trop donné à l’oreille, & que ceux de Pythagoré ont refuse l’experience du sentiment pour suivre la seule raison des nombres,\footnote{Livre sec. p. 56.},
‘Il est aysé de conclure que la division d’Aristoxene est la plus facile, & qu’elle n’a nul intervalle qui ne se mette en practique,\footnote{Livre sec. p. 61.},

‘The disciples of Aristoxenus followed mainly their hearing, and those of Pythagoras refused to listen to their feelings and followed only the reason of the figures.’

‘It is easy to conclude that the tuning of Aristoxenus is the easiest one, and there are no intervals which could not be used.’

In Proposition VII Mersenne showed different methods to divide the octave in 12 equal half-tones.

‘Ceux qui desirent d’autre manieres pour diviser l’Octave, & le manche du Luth, & des Violes en 12 demi-tons esgaux, peuvent voir Zarlino\footnote{Au 4. Livre de son Supplement, chapitre 30.} & Salinas son contemporain\footnote{In his 3. Livre, chapitre 31} se sorte qu’il y a pres de 60 ans que l’invention des demy-tons esgaux d’Aristoxene a esté renouvellee par ces deux Musiciens,\footnote{Livre sec. p. 70.},’
‘Who want other methods to divide the octave and the neck of the lutes and viols in 12 equal-halftones (notes) can read Zarlino and his contemporary Salinas, who re-invented the equal half-tones accordingly to Aristoxenus almost 60 years ago.’

Finally we need to remark that the table of Mersenne’s calculated equal temperament, which is shown in Mersenne’s engraving of the lute and theorbo, contains minor misprints and/or miscalculations.

**Temperament de l’Epinette**

On page 109 Mersenne wrote that one started with the $F$, the notes with $d$ on top should be lowered a bit, the notes under which a $f$ was written should be a bit higher, and to make the corresponding fifths a bit narrow. The octaves should be pure; the last interval being G sharp/flat.

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48 *Livre troisième des instruments à chordes: qui est semblable à celuy de l’orgue*, (which is the same as for the organ) p. 104
Temperament des orgues\textsuperscript{49}

In his Proposition XV: \textit{Expliquer toutes les especes de Diapaisons} (Explanation of all kinds of diapasons) Mersenne gave a table with different tuning systems which contained a practical table for equal temperament which could be used to divide a monochord (see appendix 5).

Mersenne explained in his \textit{Proposition XVI} that a perfect diapason\textsuperscript{50} needed 32, 27, 25 or at least 19 different keys in an octave. But normal organs only had 12 keys in an octave and needed a ‘temperament’\textsuperscript{51}. He gives several examples of keyboards with more than 12 keys in an octave, but writes in proposition XXIX:

\textit{Tout ce que nous avons dit jusques icy ne sert rien pour la pratique des Orgues…c’est pourquoi il faut expliquer toutes les methods possibles, ou du moins les meilleures manieres dont on use pour accorder tous les jeux de l’Orgue}\textsuperscript{52}.

‘Everything we said before is not helpful in practise for the organ…this is why we shall explain the possible methods, at least the best ways to tune the registers of the organ.’

First method

On page 364, which unfortunately contains many misprints, Mersenne described the first method. He gives the following musical example (we corrected some misprints):

According to the text, the notes with ﹙ on top need to be lowered and the notes above – need to be tuned a bit higher so as to make the fifths ¼ comma narrow\textsuperscript{53} which in turn results in a meantone temperament with ended with the wolf G sharp/E flat.

On page 367 Mersenne gave extra information on how narrow the fifths should be:

\textit{Car ils doivent trembler vingt fois dans l’espace d’une mesure, pour ester parfaitement temperez.’}

\textsuperscript{49} Livre sixieme des orgues.
\textsuperscript{50} With intervals which are completely pure.
\textsuperscript{51} Which means that most intervals should be a bit out of tune in order to close the circle of fifths.
\textsuperscript{52} Page 363.
\textsuperscript{53} See page 366.
‘They need to beat 20 times during the time of a measure, to be tempered perfectly.’

Second method

‘Dans la 2. Maniere dont on use pour accorder, on prend C pour fundament’ but ‘on peut commencer par telle marche que l’on veut’.

‘In the second way one uses to tune, we take the C as first note, but it is possible to start on any key you like.’

With the following description:

1. Tune first
   the octave 12-24 (C-C)
   and then
   the fifth 12-19 (C-G)
2. The octave 19-31 (G-G)
   &
   the fifth 19-26 (G-D)
3. The octave 26-14 (D-D)
   &
   the fifth 14-21 (D-A)
4. The octave 21-33 (A-A)
   &
   the fifth 21-28 (A-E)
5. The octave 28-16 (E-E)
   &
   the fifth 16-25 (E-C sharp)

E-C sharp is not a fifth. Part of the text is missing: tuning the fifths E-B, B-F sharp and F sharp-C sharp, from now on everything is okay:

6. The octave 25-13 (Cis-Cis)
   &
   the fifth 13-20 (Cis-Gis)
7. The octave 20-8 (Gis-Gis)
   &
   the fifth 8-15 (Gis-Dis)
8. The octave 15-27 (Dis-Dis)
   &
   the fifth 15-22 (Dis-Ais/Bes)
9. The octave 22-10 (Ais/Bes-Ais/Bes)
   &
   the fifth 10-17 (Ais/Bes-F)
10. And at last
    the octave 17-5 (F-F)

This is Mersenne’s second way of tuning the organ: to tune equal temperament by making all the fifths a bit narrow, without a wolf fifth. Mersenne explained how the fifths needed to be narrowed in equal temperament:

‘La Quinte bat une fois dans chaque seconde minute, lors que la Quinte est temperée comme il faut, tant sur l’Orgue que sur l’Epinette, au lieu quand elle est juste, elle ne bat plus.’

‘The fifth beats once a second, when the fifth is narrowed as it should be, for the organ as well as for the harpsichord, in such way that if it (the fifth) is pure, it doesn’t beat at all.’

At the end of his Livre sixiesme des Orgues we find Proposition XLV we find:

54 Nouvelles Observations, p. 20.
‘Deux moyennes continuellement proportionnelles, pour diviser le Diaspon des Orgues en douze demitons esgaux.\textsuperscript{55},’

‘Two methods in continuous proportion to divide the diapason (tuning) of organs in 12 equal half-tones (notes).’

The Aristoxenus way of tuning equal temperament made it possible for the French composer Jean-Nicolas Geoffroy (1633-1694)\textsuperscript{56} to write his Livre des pièces de clavecin which systematically explored all major and minor keys.

In 1643 Jean Denis published his Traité de l’accord de l’espinette. A second edition appeared in 1650. The little book starts with a table in which he explained the differences between smaller and bigger half-tones, resulting in ‘good’ and ‘bad’ major and minor thirds. Denis’ tuning system can be used to tune equal temperament, mean-tone and everything in-between: tuning the fifths a bit narrow, and use thirds to control if the tuning is ‘right’. About the size of the fifths he used the following, very carefully formulated sentence:

‘Je ne desire point parler de la Theorie, mais seulement de la Practique & usage. Et comme nous accordons l’Espinette dans la perfection…nous baissons toutes les quintes d’un poinct, & telle sorte que la quinte paraist etre encore bonne, quoy qu’elle ne sont pas juste, & sur la quantite des quintes qui sont douze en tout…les baissons toutes d’un poinct…i l faut douze poincts, qui est la difference de la premiere à la derniere, & toutes les quintes doivent ester temperées esgallement, & toutes pareilles.’

‘I really don’t want to speak about the theory, but only about the practice in use. And when we tune the harpsichord perfectly…we diminish all the fifths with a ‘poinct’ in such a way that the fifth sounds still pure, although they are not pure, and the number of fifths in total is 12…we narrow all of them with a ‘poinct’… the difference between the first and last fifth is 12 poincts, and all the fifths needs to be narrowed equally and all the same.’\textsuperscript{57}

According to the law of mathematics: if we narrow 12 fifths, which are 12 ‘poincts’ too wide, with 1 ‘poinct’, we will get equal temperament. But reading the rest of his short description which contains some errors and ambiguities, gives the impression that Denis’ possibly wanted to tune something between mean tone and equal temperament.

It took more than a century before equal temperament became generally accepted in France. Rameau converted to equal temperament, whereas F. Couperin did not want to use it\textsuperscript{58}.

In 1753 Michel Corrette (1707-1795) published his Le Maitre de clavecin and in the chapter on how to tune the harpsichord and organ he gave several possibilities. He mentioned Mersenne and equal temperament, a wrong interpretation of Mersenne’s mean-tone temperament, but proposed a new system as well\textsuperscript{59}, an unequal temperament which would later be criticised by the German, Marpurg, as we shall discuss further on, since it was, according to the latter, being (quote) ‘against nature’.

\textsuperscript{55} Nouv. Obs. p. 408.
\textsuperscript{56} http://en.wikipedia.org/wiki/Jean-Nicolas_Geoffroy
\textsuperscript{57} A point is the smallest measurement used to divide a monochord: ‘Le pouce était traditionnellement divisé en 12 lignes, puis en 144 points.’ The inch is traditionally divided in 12 ‘lignes’ and then in 144 ‘points’.
\textsuperscript{58} Rousseau in the article Tempérament in the Encyclopédie Méthodique: Musique.
\textsuperscript{59} 8 fifths a ¼ narrow; two fifths more in tune than the preceding ones; and one fifth too wide.
5. 17th century Germany

Michael Praetorius (1571-1621) published his famous Syntagma Musicum in three parts between 1614 and 1620. In the second part, chapter XXV, Organographia he described the ‘monochordum’ and its use. The tuning which became known as the Praetorius’ temperament is described in the second part, in the chapter headed: ‘Wie man ein Regal, Clavicymbel, und dergleichen Instrument vor sich selbst accordieren und rein stimmen könne.’ (How to tune a regal, harpsichord and similar instruments properly). He writes that all octaves need to be pure, as well as the major thirds and minor sixths. The fifths need to be narrowed, the fourths too wide. As a consequence, the minor thirds were a bit out of tune, as well as the major sixths. He took his time to explain three different ways of tuning mean-tone which culminated with the following scheme:

The method of this system is explained on page 152 of the second part.

At the same time, calculations with logarithms were also being applied in Germany by 1630. It was Johann Faulhaber (1580-1635) who brought the logarithms to Germany and he produced the first publication of Henry Briggs logarithm tables in Germany. Faulhaber, at that time stayed in the German city Ulm, used logarithms to calculate equal temperament. The following list by Faulhaber, used to divide a monochord for equal temperament, was reprinted in Neidhardt’s Sectio canonis harmonici (1724):

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60 Ingenieurs-Schul Erster Theyl, Ulm (1630).
61 Page 24
Wolfgang Casper Printz (1641-1717) wrote his *Phrynis Mitilenaeus*\(^6^2\), with Parts I, II and III combined and re-printed in 1696. He described the use of a monochord, but according to Printz the best tuners were those:

‘welche am allerersten c und e ganz rein/ hernach die Quinten/ cg, gd, da, niedrig schwebend/ daß man es kaum mercke/ zusammen stimmen/ und so sie dan befinden/ daß die Quinta ae weder mehr noch weniger schwebe/ als die vorgemeldeten/ welche auch gleichmaßig schweben müssen/ so wird die Temperatur ihre ziemliche Richtigkeit haben.’\(^6^3\)

‘who start tuning c and e totally pure and afterwards tune the fifths cg ,gd, da, beating narrow in such a way that it is almost unnoticeable, and when they find out that the fifth ae has no more nor less beating as the former ones, which need to beat evenly, then the temperament will be rather right’.

‘Ferner stimmen sie die erwehneten Clavium-Octaven/ und dann die Tertias majors, fs zum d, es zum g, gs zum e, f zum a, as zum fs (sic), h zum g, as zum a (sic), b zum d und ds zum h rein/ und endlich ihre Octaven auch ganz rein zusammen/ so wird das ganze Instrument seine möglichste Reinigkeit haben.’

‘Then tune the mentioned keys in octaves, and then the major thirds fis from d, es from g, gis from e, f from a, as from fis (misprint), b from g, as from, es from a (another

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\(^{62}\) The title refers to Phrynis, a classical Greek cithara player from Mytilene who was a key figure in ‘modern music’.

\(^{63}\) Phrynis Mitilenaei, page 68.
misprint), bes from d and dis from b pure/ and at last their octaves also totally pure/ and the whole instrument will be tuned as pure as possible."

At the end of his book is a list of Corrigenda, but the little misprints in tuning meantone are not in his list. Instead of correcting them he writes at the end: ‘Die übrigen Fehler wolle der günstige Leser für sich selbst zu corrigieren belieben.’ ‘Please correct the rest of the mistakes yourself.’

Printz was convinced that this mean-tone system, with 100% pure thirds, was the only correct temperament, but he also knew that there were alternative mean-tone systems. He mentioned that Larzinus divided the comma in seven equal parts, but in this tuning as well as in other alternate mean-tone systems, the thirds were not completely pure. Printz condemned, with typical baroque exaggeration, all the temperaments in which the thirds were not 100% pure (quote): ‘daß wie weniger der Quint genommen wird…je unreiner und greulicher die Tertiae und Sextae.’ With the result that this temperaments (quote) ‘dem Gehöre Mäuse=Dreck für Pfeffer verkaufen.’ Translated: ‘the less is taken from the fifth, the more out of tune are the thirds and sixths…which is like selling mouse droppings for peppercorns.’

It would not be surprising that Printz was totally against a temperament in which it was possible to modulate and transpose in all 24 keys. He wondered (quote): ‘Was haben denn solche transponir=süchtige Leuten für Nutzen von ihren übermäßigen transponiren? Sie…verderben muthwilliger Weise Finem Musices Internum, nehmlich Harmonian’. Translated: ‘What do such people who are addicted to modulations when they modulate so excessive achieve? They ... spoil the essential purpose of the music deliberately, namely the harmonies.’

Printz disagreed with some of his colleagues:

‘Ich habe offt nachgesonnen/ warum doch etliche von denen neuen Componisten so wunderlich Transponiren/ aber habe keine andere Ursache ergründen können/ als/ weil sie sonst nichts erfinden können/ damit sie also etwas neues und ungewöhnliches auff die Bahne brächt/ und bey denen Unverständigen dadurch in Admiration und Ansehen zu kommen/ welches ihnen aber bey verständigen Musicis wenig Ruhm bringe/ sondern sie vielmehr überzeuget/ daß sie die Music ex Fundamento nicht verstehen.’

Translated: ‘I have often wondered why some of the modern composers modulate so weirdly, but I have no other solution than that they want to have something new and unusable in staging and thus get admiration and respect from stupid people, in a way that brings little fame by sensible musicians, but to them (sensiblele musicians) convincingly show that they know nothing of the founding principles of the music.’

In other words, Printz tried to stop the use of modulating out of the mean-tone temperament and the use of all tonalities, but did not succeed.

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64 Interesting is that Prinz gives the possibility to tune as and gis as well, dis and es as well.
65 And diminished the fifths with 2/7 part of a comma, Phr. Mit. page 69
66 Page 89
67 Page 78
68 Page 74
J.G. Meckenheuser (1666-1727) in his *Musikalische Temperatur* (1727) confirmed that the 1/12 comma narrowed equal temperament was already used in the German city Ulm during the first half of the 17th century and around the second half of the 17th century in Hamburg as well.

During the 17th century in Germany there was some reluctance to convert from the meantone system to the method of equal temperament. According to Mattheson and Meckenheuser, equal temperament was occasionally used in the 17th century, but it remained an exception.

As one will read in the chapter on Werckmeister, equal temperament was already reintroduced in Germany by 1691 by Werckmeister himself and more and more composers preferred modulating/transposing with several composers starting to use all 24 tonalities.

6. 17th and 18th century England

John Wallis (1616-1703) was a mathematician who also dealt with equal temperament. He wrote letters in 1664 to the German Henry Oldenburg concerning tuning and approved of equal temperament that was being used in England's organs. In one of his letters Wallis proposed improvements to the tonal scales by employing the theory of proportions in the determination of intervals. Oldenburg (c. 1619 – 1677) was a German theologian known as a diplomat and a natural philosopher. He was one of the foremost intelligencers of Europe of the 17th century, with a network of correspondents including Marin Mersenne (of whom we already have discussed). In the 17th century, music was part of the mathematics field and all the important scientists knew each other or each other’s work. John Wallis was not forgotten in the first half of the 18th century in Germany: Johann Fux quoted Wallis in his famous *Gradus at Parnassum* (1725).

Gottfried Keller (Godfrey) (1650-1704) was a German who relocated to England at the end of the 17th century and wrote a *Method for attaining to play a Thorough Bass* in which instructions were given to tune a harpsichord or spinet. The book was published shortly after his death in 1704.

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69 Page 10
70 Meckenheuser *Musikalische Temperatur* p. 10; Mattheson *Critica Musica* (1722) p. 11, *Critica Musica* (1725) p. 162.
71 In his *Musikalische Temperatur*
73 http://emlo.bodleian.ox.ac.uk/profile/work/8527f90c-1709-448d-a23a-f2e113f89e42?sort=date-a&rows=50&rec=oldenburg&aut=wallis&baseurl=/forms/advanced&start=4&type=advanced&numFound=175
75 http://emlo.bodleian.ox.ac.uk/profile/work/9a600eb0-bcb7-4269-a6b4-c3a3719571ec?sort=date-a&rows=50&rec=oldenburg&aut=wallis&baseurl=/forms/advanced&start=6&type=advanced&numFound=175
76 In the German translation by Mizler page 2.
Observe that *all* sharp thirds must be as sharp as ye Ear will permit and *all* Fifths as flat as the Ear will permit

Keller's tuning method was reprinted a number of times⁷⁷. William Holder added Keller’s book and tuning method as an appendix in his *Treatise of Harmony* (1731).

Alexander Malcolm (1685-1763) in his *Treatise of musick* written in 1721 wrote that there were different views in his time on which temperament to apply. Some harpsichordists tuned the fifths (according to Malcolm): *‘as perfectly Concord as their ear can judge’*⁷⁸… which indeed makes a great many Errors in the other intervals of 3d and 6th…others that affect a greater Nicety pretend to diminish all the 5ths, and make

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⁷⁷ Among others in a French translation in 1753 in: *Le maître de clavecin* of Michel Corrette, page 86

⁷⁸ Pythagorean tuning
them deficient about a Quarter of a comma\textsuperscript{79}, in order to make the Errors in the rest smaller and less sensible.’

In the same chapter Malcolm quoted the 17\textsuperscript{th} century mathematician Wallis: ‘That the (organ) pipes within an octave…shall be in continual Proportion, where it comes to pass that each Pipe doth not express its proper Sound, but something varying from it, which is called Bearing; and this, says he (Wallis) is an imperfection in this noble instrument. Again, he says, That the Semitones being all made equal\textsuperscript{80}.’

William Tans'ur (1706-1783) in his New Musical Grammar of 1746 described how he tuned harpsichords and organs:

‘Observe, to Tune all Sharp-Thirds, as sharp as the Ear will admit; and also all 5ths bearing; that is as flat as possible: which will render your Musick the more Grand and Harmonious: And often, by way of Trial, touch Unison, Third, Fifth, and Eight altogether; and also Unison, Fourth, and Sixth: and lastly, if every Octave of your Keys, both Proper-Notes and Semitones, sound perfect Eights to each other, then you may conclude, that your Instrument is in perfect Tune\textsuperscript{81}.

7. End 17th century and 18th century Germany

During the 17\textsuperscript{th} century in Germany there was some reluctance to convert from the meantone system to the method of equal temperament. Accordingly, to Mattheson and Meckenbeuser, equal temperament was occasionally used on organs and harpsichords in the 17\textsuperscript{th} century\textsuperscript{82}, but they were the exception.

Andreas Werckmeister (1645-1706) proposed a circular system with a closed circle of fifths without a wolf fifth. Werckmeister’s publications could be viewed in three time-frames\textsuperscript{83}.

1. From 1681-1691 Werckmeister was devoted to unequal temperaments, in which thirds were more pure than in equal temperament, with the disadvantage that the fifths and fourths were rather impure and as second disadvantage was, that some thirds were rather pure and other thirds were much more out of tune. Some modern players like these variations in purity of the thirds, but Werckmeister got criticised and improved his tuning systems.

2. From 1697-1698 he was concerned with the use of most or all keys in performance and therefore Werckmeister proposed an almost equal temperament and equal temperament if one used all tonalities.

\textsuperscript{79} Meantone temperament
\textsuperscript{80} Semi tones equal in proportion produces equal temperament, but there are difficulties in interpreting Malcolm’s text properly based on whether Malcolm would have preferred equal temperament or not.
\textsuperscript{81} Page 70-73
\textsuperscript{82} Meckenbeuser Musikalische Temperatur p. 10; Mattheson Critica Musica (1722) p. 11, Critica Musica (1725) p. 162.
\textsuperscript{83} The Dutch musicologist Rudolf Rasch mentioned this already in 1985 in his article: Does ‘Well-Tempered’ Mean ‘Equal-Tempered?’ in Williams’ compilations of articles: Bach, Handel, Scarlatti; Tercentenary Essays.
3. At the end of his life, 1702-1707 he was a proponent of equal temperament, since it made possible unlimited modulations and transposition and equal temperament became his first choice.

In 1691 Werckmeister published his *Musicalische Temperatur*. Similar to his predecessors, Werckmeister made many calculations with the help of a monochord. He even encouraged his readers to build their own monochord. In the subsequent chapters he gave different ways to divide the comma on the circle of fifths, including the now familiar Werckmeister III temperament.

![Diagram of the Werckmeister III temperament]

But the applicability of this temperament by Werckmeister did not last long.

In 1698 Werckmeister published: *Anmerkungen und Regeln (Observations and Rules on how to play the basso continuo)*. In this treatise Werckmeister acknowledges the existence of equal temperament and writes: “in the meantime one can use all 12 keys on a tempered keyboard”.

In the last chapter of Werckmeister's 1698 treatise one finds: a 'short instruction on how to tune a keyboard in a good temperament'. Here he distanced himself from his previous temperaments and proposed a new compromise: an almost equal temperament in which all keys were usable and the most used tonalities being somewhat purer than in equal temperament: hence an almost equal temperament. And he adds (quote Werckmeister):

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84 Page 77  
85 Page 22-23  
86 7 fifths too small, a fifth nearly pure, two fifths somewhat big and the last somewhat too big or pure - a temperament that he will later get much criticism for.
'All fifths can hover down a 1/12 comma (with the top note in relation to the bottom note)...so that it is tolerable if one wants to play the entire keyboard and for all songs (compositions) in all tonalities.'

However, Werckmeister changed his mind on temperament again in his treatise *Musikalische Paradoxal-Discourse* issued in 1707, published a year after his death. Here he wrote:

'We know that in a temperament in which all the fifths are 1/12 comma narrowed...and an accurate ear can achieve and tune this, than we will get a 'wohl temperierte Harmonia' (well tempered Harmony) in the whole circle (of fifths) and in all tonalities...'

Thus Werckmeister had converted to equal temperament. Of other temperaments he wrote that:

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87 Page 63-64
88 Page 110
'If one makes a number of thirds too pure, and then this is an insult to the other harmonies and also for the fifths'.  

Consequently he became, towards the end of his life, a true proponent of the equal temperament, in which all fifths are tempered by a 1/12 comma.

Werckmeister explained why the equal temperament was the best: The Lutheran orthodoxy and the after effects of the humanism formed the framework of thought for Werckmeister and also for J.S. Bach. Werckmeister quotes Johan Kepler’s *Mysterio Cosmographico*: ‘God is the author and creator of harmony’.

In chapter XXIII on *Radikal-harmonischen Zahlen*

Werckmeister explained that harmony could be expressed in its entirety in super particular numbers. Different particular numbers refer to (in order of importance) God, Jesus Christ, humans, animals and the Devil (Lucifer). The same description is given by Fux in his *Gradus ad Parnassum*. (A famous example of using the super particular 1:2 as metaphor for Jesus Christ we find in Bach’s St Matthew Passion, the 2nd part, number 67, recitative-chorus section, ‘I am the sun of God’ is sung and played with only unisons and octaves.)

According to Werckmeister’s ideas about super particulars, the octave should be perfectly pure, and the smaller the intervals (in the order fifth, fourth, major third, minor third etc.) the more they needed to be out of tune. Equal temperament became the first choice on theological grounds for Werckmeister but an almost equal temperament was acceptable to him.

The reason for his not introducing equal temperament in his first book on tuning is explained by Werckmeister as follow:

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89 Page 108
90 Page 14
91 See Wikipedia.org/wiki/superparticular_number
92 Page 35
93 The unison is a metaphor for God, the super particular 1:2 with the octave is a metaphor for Jesus Christ, see page 92 and 102
94 See the St Matthew Passion, end of the Coro: *Andern hat er geholfen*. The text: *Ich bin Gottes Sohn.*
Firstly Werckmeister III was an improvement on ¼ comma meantone and (quote Werckmeister) 'not every organist used all the tonalities.'

Secondly:

‘You can’t demolish a building with strong foundations in one go’.95

And lastly, in the last chapter of his 1707 treatise he lists the ¼ mean tone tunings, but described them as (quote): 'die unrichtigen Temperatur' (the wrong temperaments).96

8. Introduction of the equal temperament (gleich-schwebende Temperatur) in Germany in the 18th century

In 1706 Johann George Neidhardt (1685-1739) published his Beste und leichteste Temperatur des Monochordi97 (Best and easiest Temperament of the Monochord).

He gave the following example:

He explained that the upper c’’’’’’ (which one gets when one tunes 12 consecutive pure fifths) is not the same as the lower c’’’’’’ (which one gets when one tunes 7 pure octaves): the difference being the Pythagorean comma. Neidhardt completes the circle of fifths by tempering all 12 fifths by a reduction of a 1/12 comma (as we have already seen described by Mersenne and others).

However, there was a novelty: for the first time, the German term gleich-schwebend (equal temperament) is used by Neidhardt in 1706. In chapter VII of his method Neidhardt writes how one can improve the diatonic-chromatic scale by applying a 'gleich-schwebende' (equal beating) temperament and Neidhardt quotes Werckmeister: ‘Who demands a temperament in which all consonants are equal, he makes all fifths 1/12 comma narrow’.98

95 Page 113
96 Page 109
97 des monochordi, vermittelt welcher das heutiges Tages brauchliche Genius Diatonico-chromaticum also eingerichtet wird/ dass alle intervalle, nach gehöriger proportion, einerly Schwebung überkommen/und sich daher die modi regulares in alle und jede Claves, in einer angenehmen Gleichheit/ transponieren lassen.
98 Page 40
In rejection of the old Werckmeister temperaments, Neidhardt entitled the following chapter: 'a new diatonic-chromatic-enharmonic scale that is subdivided in accordance with the equal temperament.' He calculated a ‘gleich-schwebende’ temperament which ended up being used by most musicians.

At the end of his book he gives a practical rule to tune: every fifth 1/12 narrow, and use thirds to cheque if there is no mistake made.

‘Compare the (last) c’ with the (first) c. Well done if the octave is pure. If not, then you need to go back and find out where you made a mistake... Finally you tune the rest of the keys with pure octaves.’

From the early 18th century onwards, a large number of publications appear in Germany about equal temperament, the most notable authors of which being:

Christoph Albert Sinn (1680-1729) who published his *Musicalische Temperatura* (1717) in which he used proportions, roots and logarithms to calculate equal temperament.

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99 Some thirds, especially Cis-F, fis-B and Gis-c are unbearably too big, some minor thirds eg. C-Dis, Gis-B are unbearably too small’, something that Werckmeister himself already admit to.

100 Page 42

101 Page 103

102 *Die aus mathematischen Gründen richtig gestellte Musikalische Temperatura Practica.*
**Johann Joseph Fux** (1660-1741). The only surviving book of Bach's personal library of theoretical works, with Bach's ownership note, is the Latin edition of Fux's *Gradus ad Parnassum* from the year 1725\(^{103}\).

Fux' *Gradus ad Parnassus* was translated into German by Mitzler in 1742. Fux explains that intervals in exact mathematical proportions result in smaller and bigger half tones. He mentions that some organists added extra keys: *split halves* to use smaller and bigger half tones. But adding extra keys on a keyboard was problematic and for this reason they divided every note in (quote): ‘*zwei gleiche Theilen’* (two equal parts), resulting in equal temperament. He continues:

‘*Da man erfahren, daß solches in Zahlen nicht angeht, ist das Ohr zu hülfe genommen worden, indem man von dem einem Theil einem fast gar nicht mercklichen Theil weggenommen, und dem andern zugeseßet*’

‘*Because experience told us that one cannot do this by means of figures, the help of the ear was called in to help, by taking away an almost non-detectable amount of one note and added it to the others.*’

This is the Aristoxenus’ way of tuning equal temperament by ear and not by the Pythagorean calculating way\(^{104}\).

Fux writes that in old organs in mean-tone it happened that ignorant organists modulated to keys which resulted in (quote) ‘*ein verdrießliches und rauhes Geheule’* (morose and harsh howling).

The next chapter: *Vom heutigen musikalischen System* (*Of today's musical system*) starts (quote): ‘*Da man die Ungleichheit der Tone und halben Tone aufgehoben.*’ (*‘We gave up the un-equal tones and half tones.’*), but he mentions that a choir, without organ accompaniment, still uses pure intonation\(^{105}\).

**Johann Mattheson** (1681-1764) wrote his *Von der Musikalischen Thone Eigenschaft und Würckung in Ausdrückung der Affecten*\(^{106}\) (*About the different properties and expression of tonalities and the effect on the affections*) in his *Neu-Eröffnete Orchestre* (*1713*). Mattheson wrote about the differences of *Tertia Minore* and *Tertia Majore* and about the different Modi in the antiquity. But Mattheson was a progressive musician and theoretician who promoted Werckmeister’s equal temperament.


\(^{103}\) Christoph Wolff: *Johann Sebastian Bach* second edition 2000 page 359.
\(^{104}\) *Gradus ad Parnassus* page 52.
\(^{105}\) Page
\(^{106}\) Page 231.
Which means: An exemplary organists test in articles which by means that we employ 24 easy and the same amount of a bit more difficult examples in all tonalities, so that anyone who can play these 48 exercises and applies the content properly, themselves may glorify: He is a master in accompanying.

Note that Mattheson (who promoted equal temperament) published his 48 exercises through all tonalities 9 years before Bach compiled his Well-Tempered Clavier.

Many organs at that time were at ‘Chor-ton’ (chorus/choir pitch) which is higher in pitch than ‘Kammer-ton’ (chamber pitch) of the other instruments. The higher the pitch of church bells, carillons and metal organ pipes, the less precious metal was needed to make them. In order to play with other instruments, the organists needed to transpose their parts, which could cause real problems using organs in mean-tone. In his Forschende Orchestra (1721) Mattheson quotes Neidhardt about unwanted transpositions:

‘daß ich mit Neidhardts Worten rede: Mann betrachte mir doch den greuel/ wenn manchesmal Instrumente/ die im Kammer-Ton stehen/ andere accompagniren sollen/ die da Chor-tönig sind/ dabey entweder diese oder jene transponirt werden müssen; klinkt es anders/ als wenn der Componist oder Cantor ‘den Zanck der Hunde über den Corper der abgestürzten Jesabel’ hätte vorstellen wollen?’

‘to use Neidhardt’s words: consider the abomination when instruments in chamber-tone accompany other instruments in chorus-tone and need to transpose; it sounds as if the composer or cantor tried to’ represent the winning of dogs by the fallen down Jezebel.'

Accordingly to Mattheson it was a pity that the differences of characteristics of musical keys disappeared in equal temperament, but for practical reason more and more organists started to use equal temperament. J.S. Bach did not seem to have serious problems by transposing old compositions when he reused them.

In his Critica Musica of 1722 Mattheson published a division for a monochord. One can use the description as a model, by cutting it in pieces, gluing them together and using the design as scale for a monochord which reflects Büßmler’s equal temperament:

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107 J.P.A Fischer: Verhandeling van de klokken en het klokke-spel (1738), page 4
"Aangaande de groote der klokken, so is sulks onbepaeld, die kan elk laeten maeken, na dat'et met syn beurs overeen komt." The size of the clocks can be as large as ones wallet allows it.
108 Neidhardt’s Beste und leichteste Temperatur (1706), page 40
109 Neidhardt quotes the Bible: Book of Kings
110 For example the famous b minor flute sonata which exists in an earlier version in g minor.
111 Crit. Mus. 1722, page 52
112 Another German 18th century writer on equal temperament.
113 Meckenuhser criticised this drawing, not because it is equal temperament, but because one of the figures is wrong: see Meckenuhser’s Musicalische Temperatur, page 27. Büßmler’s d is 2963.28 and should be accordingly to Meckenuhser: 2996.6762 and Meckenuhser is right, Büßmler’s b is too high. The rest of the figures are almost identical with the result of Meckenuhser’s calculation of equal temperament.
In his *Beschützte Orchestre* of 1727, Mattheson declares the equal temperament calculated by Neidhardt as the best. And in 1727 Mattheson promotes Meckenheuser’s equal temperament as well.

Finally, in his *Grosse General-Baß-Schule* (1731) Mattheson writes that most organs are still in unequal temperament but goes on saying (quote): ‘personally I believe that the equal temperament is the best’.

**Johann Georg Meckenheuser** (in Bach’s time a well-known musicologist) published *Die sogenannte allerneueste musicalische Temperatur* (1727). He quotes the Dutchmen Simon Stevin and Dirck Rembrandt van Nierop114 followed by a description of ‘12 proportional equal intervals’ which is approved by Johann Mattheson115: equal temperament with fifths beating in proportion.

**Lorenz Mizler** (1711-1778) is mentioned in Forkel’s *Allgemeine Literatur* under the chapter: *Histoisch-kritische Zeitschriften*. Mizler published the first part of his *Neu=eröfnete musikalische Bibliothek* in 1739. According to Forkel, Mizler strength lay in his ability to summarise books and literary annotations.116

Mizler gives lots of information about the music life of his time. In Part I he repeats what we have read of the other authors so far, namely that there was a battle between ‘Aristoxenern und Pythagoräern/ wovon diese alles mit Zahlen, jene aber durch das Gehöre beweisen wollten,’ Translated: ‘The followers of Aristoxenus and those of Pythagoras, the first who wanted to tune everything by ear, whilst the latter wanted to calculate everything’

114 Page 10
115 More authors describing (equal) temperament can be found in Gruber’s *Litteratur der Musik* (1792) page 25 e.v., in Forkel’s *Allgemeine Litteratur der Musik* (1792) pages 250 e.v. and in Sulzer’s *Allgemeine Theorie der schöne Künste*, deel IV page 466 e.v. and 520 e.v.
116 *Allgemeine Litteratur der Musik*, page 466
Mizler describes that ‘Subsemitonia’, i.e. the difference in small and large half tones in the unequal beating temperament, is not needed anymore in the new improved temperament. He does not calculate anything, but refers to his German colleagues. As we have seen they were proponents of equal temperament, namely Bümler, Werckmeister, Neidgardt and even Hensling\(^{117}\) and the French theoretician, Sauveur.\(^{118}\)

**Georg Andreas Sorge** (1703-1778) published his *Clavier Übung* in two parts, 24 preludes for organ or clavichord (1739–42) that passes through all keys. In his *1744 Anweisung zur Stimmung u. Temperatur* he calculates the equal temperament with in proportional beating fifths. Sorge even went as far as to build monochords with subdivisions for equal temperament so that harpsichords could be tuned with them. He sold them on the Leipzig Market in 1750\(^{119}\).

In 1758 Sorge publishes his *Zuverläβige Anweisung Claviere und Orgeln zu stimmen*. This book arises in a form of a rebuttal to criticism by Fritz (see further) as to the method of tuning described by Sorge in his 1744 book. But about Fritz’s tuning he says (quote Sorge)\(^{120}\): ‘Ich will aber billiger seyn als er, und zugeben daβ seine Stimmung gut seyn könne, wenn sie auch gleich nicht völlig rational=gleich wäre...und bemühet ist, so viel dem Gehör möglich, alle 24 Tonarten in gleicher Reinigkeit darzustellen\(^{121}\).’

‘I will be more reasonable than he is, and admit that his tuning could be good, although not exactly ‘rational=gleich’ (equal temperament with in proportion beating fifths)... and he takes care to make all 24 tonalities equally pure by ear.’

**Jacob Adlung** (1699-1762) in 1758 wrote his: *Anleitung zu der musikalischen Gelährtheit (Instructions on acquiring musical erudition)*. The book gives much theoretical and practical information on music, in particular for keyboard players, organ and other instrument makers. The foreword is by **Johann Ernst Bach** (1722-1777), a distant cousin and pupil of J.S. Bach.

In many places, Adlung writes about the ‘die gleichschwebende Temperatur’ (equal temperament)\(^{122}\) mentioning that (quote) ‘since the introduction of the equal temperament it does not matter whether one plays in C or C-sharp, F or F-sharp etc.’\(^{123}\) According to Adlung it was Andreas Werckmeister who introduced ‘die gleichschwebende Temperatur in Germany’\(^{124}\): ‘Werckmeister verdient allhier vor vielen gemerckt zu werden. Er hat in seinen Schriften die gleichschwebende temperatur vorgeschlagen, wie Neidhardt selbst gestehet.’

\(^{117}\) Conrad Hensling (Hänsling) divides the octave in 50 parts, according to Mattheson’s *Crit.Mus*, part I, page 51.

\(^{118}\) Page 55.

\(^{119}\) Adlung, *Zur musikalischen Gelahrheit* (1758): Sorge machte sie (monochorde) 2 Schuhe lang mit einem Futteral zum Verkauf. (Sorge made his monochord 2 feet long and sold them at the market.)

\(^{120}\) *Zuverläβige Anweisung Claviere und Orgeln zu temperieren und zu stimmen* (1758), page 9

\(^{121}\) Sorge noticed that there is a difference between his calculated equal temperament with in proportion beating fifths and Fritz’ practical equal temperament tuned by ear with equally beating fifths.

\(^{122}\) See pages 226, 236, 243, 247, 284, e.v. 306, 310, 316, 327, 329, 326

\(^{123}\) Page 226

\(^{124}\) Page 284
Werckmeister is being noticed here before many others. He has proposed the equal temperament in his writings, as Neidhardt himself admits.\(^\text{125}\)

Werckmeister mentioned equal temperament already in 1691. He proposed equal temperament in 1698 for compositions through all tonalities (24 years before J.S. Bach compiled the first half of his Well-Tempered Clavier) and at the end of Werckmeister’s life equal temperament was his first choice.

Because it was a major and costly exercise to tune them, many old organs remained tuned in meantone\(^\text{126}\). But according to Adlung several organs were already in almost equal- or equal temperament. Close to equal temperament was acceptable for Werckmeister and for Adlung as well, and equal temperament was Adlung’s preferred choice\(^\text{127}\). In the chapter ‘How to build an organ’ Adlung says that he himself always prescribes ‘the equal temperament’\(^\text{128}\).

Adlung instructs how to tune in equal temperament (quote)\(^\text{129}\): ‘an experienced tuner does not use a monochord, but makes the fifths somewhat small in such a way that the ear does not hear any beating.’

With the opportunities offered by equal temperament it became a trend to compose ‘musical circles’: pieces that modulate through all twelve tonalities. Johann David Heinichen (1683 – 1729) studied at an early age with Johann Kuhnau, Bach’s predecessor as cantor of the Thomas Kirche. Already as student Heinichen modulated through all tonalities and in his Generalbaß in der Komposition (1728) he explains how to do this\(^\text{130}\).

Composers such as Heinichen, Kellner, Mattheson and Sorge composed ‘Cirkelgänge’ in which all 24 keys were used\(^\text{131}\). Organs that were tuned in meantone sometimes had double keys: split halves. As a result, it was possible in meantone to use both D-sharp and Es-flat, A-flat and G-sharp. But Adlung mentions\(^\text{132}\) that these split keys on organs were no longer needed following the introduction of ‘the equal temperament’.

However not everyone welcomed equal temperament. Adlung writes that Neidhardt did not want to obligate anyone to use the equal temperament and so published a second book with four temperaments that would be most suitably used respectively for a ‘village’, a ‘small town’, a ‘large city’, and the last (the equal temperament) for the ‘Court’\(^\text{133}\). A village organist who plays hymns in only a limited number of keys is better off with meantone. An organist who wants to play a few solo pieces in some more varied tonalities should aim for the ‘small town temperament’, and so Neidhardt meets the various requirements of different organists.

\(^{125}\) Page 284 in Werckmeister’s Wegweiser zu der Erkenntniß der musical. Proportionen, und des Monochords Frankfurt and Leipzig 1687; and his Musikalische Temperatur 1691.

\(^{126}\) Some pipes were too short and others too long for equal temperament.

\(^{127}\) Page 327

\(^{128}\) Page 526

\(^{129}\) Page 310

\(^{130}\) Second edition, page 840-843

\(^{131}\) See Adlung’s Anleitung, from page 333 onwards

\(^{132}\) Page 243

\(^{133}\) Page 316
Adlung cites Neidhardt’s attempt to tune the organ in Jena with the help of a monochord in the equal temperament. The organist J.N. Bach, a cousin of J.S. Bach, was pitted against Neidhardt and tuned the organ by ear. According to Adlung, this experiment failed because it was not possible in practice to tune an organ pipe with the help of a monochord. Consequently, J.N. Bach’s system, using the ear, was preferred.

A similar experiment failed for the same reason with the ‘the beautiful Marienstifts organ in Niechenberg’. And around 1720 in the Netherlands, an attempt of the same sort to tune an organ with help of a monochord failed on the organ in Alkmaar.

Gottfried Silbermann (1683-1753) was a member of a family who made organs. His tuning can be found on most of our electronic tuning devices. Silbermann was famous because of the beauty of the sound of his organs but criticised on behalf of his unequal tuning.

Sorge criticized the Silbermann temperament in 1748 as we shall see in detail in Bach Dokument 575 in the second half of this article.

Adlung wrote about Silbermann in his Anleitung: ‘Er wollte durchaus die gleichschwebende (Temperatur) bey seinen Wercken nicht dulden.’

‘He refused to use equal temperament for his organs.’

Silbermann did not accept contracts, like the other organ builders which obliged him to use equal temperament.

‘Silbermann aber blieb bey seiner Stimmung; und entweder waren die ContractksSchmiede dabey nachlaßig und überliessen es ihm, als einem grossen Künstler; oder man muß allda die chromatischen Tonarten nicht so sehr lieben, wie an andern Orten.’

‘Silbermann continued to use his temperament; either the makers of the contracts were careless and left the matter to him (Silbermann) because he was a great artist; or they (the organists) did not like the chromatic keys in the same ways as they do it at other places.’

‘Er (Georg Andreas Sorge)erzählt zugleich, daß einige wackere Tonkünstler Silbermannen zugeredet, seine Stimmung anders einzurichten; allein es habe Niemand das Glück gehabt, ihn von seinem Eigensinn abzubringen.’

He (Georg Andreas Sorge who played several Silbermann organs) said at the same time that some worthy musicians tried to persuade Silbermann to adjust his tuning, but nobody was lucky enough to change his stubbornness.’

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134 Page 311
135 Johann Nicolaus Bach was the eldest son of Johann Christoph Bach. He was also the uncle of Maria Barbara Bach (J.S. Bachs first wife).
136 More details about it you will find in Adlung’s Musica mechanica organoedi (1768) p. 54-55 and Marpurg confirms the impossibility to tune an organ pipe with help of a monochord in his Mus. Temp, page 137.
137 Page 318
138 Page 320
Petri, as we shall see in Bach Dokument 868 did not have much to say about Silbermann’s temperament (quote): ‘die schönen Silbermannischen Orgeln waren falsch gestimmt.’

‘The beautiful Silbermann organs were out of tune (wrongly tuned).’

Petri wrote in the same document that because of J.S. Bach’s unexpected modulations the use of equal temperament described by Barthold Fritz became necessary.

Crucial information on how to understand Bach’s temperament and the essentials when tuning keyboard instruments by ear is given by the German instrument builder Barthold Fritz.

In 1756 Barthold Fritz (1697-1766) published a method to tune ‘Clavieren, Clavecins and Organs’ by ear. For someone who never tuned a keyboard by ear, it looks like a slightly cryptic book, but for tuners who were accustomed to tune by ear, the tuning instructions are crystal clear.

Fritz had already used this system for 40 years, well before Bach wrote the first part of the Well-tempered Clavier. His method allowed instruments to be played in all 24 keys, in which intervals were equally pure. The booklet, translated into Dutch and English, was reprinted and its instructions were used well into the 19th century, a 1780 edition being recommended by C.P.E. Bach himself. Barthold Fritz quotes Mattheson (without mentioning Mattheson’s name though) that, of our senses, hearing is the least accurate. Like Mattheson he demonstrates this by placing the bridge of a monochord a little further to the front or behind the calculated point of an interval: the eye will notice the deviation but the ear will not being able to hear the difference. An interval that is slightly too small will sound pure and so will an interval that is slightly too wide.

He describes this as the: the three degrees of purity:
• The first pure degree: an interval that is somewhat too small but not beating (and already sounds pure).
• The whole pure degree, that is perfectly pure;
• The abundant pure degree: an interval that is somewhat too wide, but no beating (and still sounds pure).

Barthold Fritz put this phenomenon to use when he tuned a harpsichord.

139 Murray Barbour: Tuning and Temperament, page 47
140 Neu eröffnete Orchestre page 287.
141 An interval which is too narrow ‘hovers’ and the more pure, the less this hovering will be. When the interval is 100% pure, there is no hovering anymore and when the interval becomes too high, the hovering starts again. This hovering is an alternation of strong and soft, comparable with vibrato on wind instruments.
142 Fritz notes that this is the case with harpsichords. On organs, if you wait long enough, you will notice some beatings, but the harpsichord tone dies out before you hear the beatings. (Onderwys om clavieren en orgels in alle 24 toonen even zuiver te stemmen, p.30)
Every fifth is tuned slightly smaller until it reaches the first pure degree (barely pure) and until there were no longer any beats\footnote{On an organ with a sustained tone intervals with one beat and even half of a beat a second will be noticeable. But on a harpsichord the tone dies out rapidly, so even if an interval is one beat different the ear will not be able to hear it, because the tone dies out before one can here the beating.}. Thus the fifth is slightly small, but effectively sounds pure. And if all the fifths were tuned slightly small in this manner, in the order of the circle of fifths, then this temperament results in equal temperament. If a tuner uses this method for the first time, he needs to practise. If the last note is too high, the fifths should be a bit narrower. If the last note is too low, the fifths are too narrow. As soon as somebody is able to end upon the same note on which one started, it is one of the easiest and fastest ways of tuning a harpsichord. Fritz system was used from the beginning of the 18\textsuperscript{th} century till after 1960 when electronic tuners came in use, and is still in use by harpsichord tuners who prefer to tune by ear.

To be clear: this method is not the same as the proportional equal temperament on electronic tuning devices. The latter temperament has proportional beating fifths, whereas all the fifths are tempered about one beat too small in Fritz’s method.

So there is a difference between the equal temperament with in-proportion beating fifths on modern tuning devices and Fritz temperament with equal beating fifths. This phenomenon is caused by a law of mathematics which mathematicians can calculate and harpsichord tuners can’t change.

In all above-mentioned publications of different writers on temperaments, we have seen that the calculated temperament was equal temperament with fifths beating in proportion, but in practice, equal temperament required equally beating fifths.

The difference between these two tunings results in the following: the thirds are notably different in the equal beating temperament. Rather, in a 100\% accurate proportionally beating equal temperament, all thirds and other intervals are 100\% the same and all tonalities sound the same.

\textbf{Johann P. Kirnberger} (1721-1783) occupies a special place in terms of tuning. In 1760 he published: \textit{Construction der gleichschwebende Temperatur}\footnote{See Forkel’s \textit{Allgemeine Litteratur der Musik} (1792) page 252 and Gruber’s \textit{Litteratur er Musik} (1792) page 27.} (\textit{Construction of equal temperament}). He also published 3 irregular circular tunings which do not matter for understanding Bach’s temperament, because they are different to the tuning that Kirnberger learnt of Bach.

Kirnberger was heavily criticised for his irregular circular tunings. Marpurg, one of the 18\textsuperscript{th} century most important musicologists specialises in temperaments in Germany, mentions that (quote): \textit{the Kirnbengers’s famous temperament was highly praised but not}...
Kirnberg's attempt to stop the advance of equal temperament failed.

**Friedrich Wilhelm Marpurg** (1718-1795) we will discuss last, because he gives a good overview of the state of temperaments in Germany during the first half of the 18th century. His *Historisch-kritische Beyträge zur Aufnahme der Musik* (1756-78) and *Versuch über die musikalische Temperatur* (1776) deals with the necessities of temperaments, and surveys different practical and theoretical tuning methods from ‘equal temperament’ and ‘almost equal temperament’ to ‘unequal temperament’.

Marpurg divided the unequal temperaments into two categories, but these (quote) ‘are of no good’. The second of the two categories is the worst because tuning with fifths which are too wide (quote) ‘is against nature’. Temperaments makes it necessary to narrow fifths, so in his view too wide fifths in a temperament is ‘against nature’.

In his *Versuch über die musikalische Temperatur* Marpurg compares and calculates all kinds of possible and impossible temperaments. From a temperament with two tempered fifths, three, four, five and temperaments with 6 tempered fifths and at the end of the book Kirnberger’s temperament.

Equal temperament was the preferred one, Marpurg giving a practical method on: ‘How to tune a keyboard in the equal temperament without the help of a monochord’.

Here he mentions Barthold Fritz who (quote) ‘differentiates regarding the fifths three degrees of purity, being the incomplete pure, the perfect or natural pure, and the abundant or too pure degree’.

Before discussing J.S. Bach’s tuning according to his contemporaries, some information on his son about Carl Philip Emanuel Bach.

In 1753 C.P.E Bach (1714-1788) published his *Versuch über die wahre Art das Clavier zu spielen*. In his book he advocates that keyboards (harpischord and forte piano) should be ’gut temperirt’ (well-tempered) by which he meant that most fifths should be made slightly small so that one hardly notices and so that one can use all 24 keys well.

But further on in the text he mentions that (quote) ‘Auf dem Claviere spielet man aus allen vier und zwantzig Ton=Arten gleich rein’ (On the Claviere one plays in all 24 tonalities equally pure.’

This description has puzzled more than one reader in our time, as the concept of most fifths being too small and all 24 keys being equally in tune is a *contradictio in terminis.*

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145 Neue Methode allerley Arten von Temperaturen (1790) Vorbericht page I
146 Historical-critical contributions of music
147 Treatise of musical temperament
148 Kritische Briefe: page 447
149 Der letzten, lauft wider die Natur
150 Über die mus. Temp. blz. 219
151 Mus. Temp. Achtzehnter Abschnitt, paragraph 161
152 ‘Er unterscheidet in Ansehung der Quinten dreierley Grade der Reine, welche er durch das erstere oder unvollkommene Reine, durch das ganz oder natürliche Reine, und durch das überflüssige oder zuviel Reine characterisiret’
153 Page 10
For readers in Bach’s time C.P.E. Bach’s intentions may have seemed clearer as both options were in use. Most fifths could be reduced so that all 24 keys would be usable, but if one were to use all 24 keys ‘gleich rein’ (equally pure) then people opted for equal temperament.

\[154\] ‘Durch diese neue Art zu temperieren sind wir weiter gekommen als vor dem, obschon die alte temperatur so beschaffen war, daß einige Ton=Arten reiner waren.’

‘The new method of tempering marks a great advance over the old, even though the latter was of such a nature that a few tonalities were more pure.’
Synopsis Part One

- **Pythagoras** calculated intervals with the help of a monochord.
- **Aristoxenus** tuned equal temperament by ear.
- **1530 Lanfranco** tuned in the Aristoxenus way equal temperament or almost equal temperament.
- **1558 Zarlino** tuned equal temperament with a mesolabium.
- **1516 Aron** used meantone temperament.
- **ca. 1605 Simon Stevin** calculated equal temperament in the Netherlands.
- **1620 Michael Praetorius** described mean-tone schemes in Germany.
- **1630 Faulhaber** introduced logarithms and calculated equal temperament in Germany.
- **1636 Mersenne** calculated and tuned meantone temperament and equal temperament by ear in France.
- **1643 Denis** tuned a kind of meantone or equal temperament by ear.
- **1664 Wallis** proposed equal temperament in England.
- **1691 Werckmeister** introduced equal temperament in Germany.
- **1696 Printz** disapproved of equal temperament in favour of mean-tone.
- **1697 Werckmeister** advised equal temperament for music in all tonalities.
- **1699 Douwes**, in the Netherlands shows how to tune using a practical method.
- **1704 Keller** tuned all the fifths narrow and all the thirds wide as possible.
- **1706 Neidhardt** introduced the term: ‘die gleich-schwebende Temperatur’ (equal temperament).
- **1707 Werckmeister** introduced the term ‘Wohltemperirte’ in his Musikalische Paradoxal-Discourse in connection with 1/12 comma narrowed equal temperament.
- **1711 Sinn: Musikalische Temperaturen**, calculated equal temperament.
- **1719 and onwards Johann Mattheson** promoted equal temperament.
- **1721 Malcolm** described the Pythagorean tuning, meantone and equal temperament.
- **1725 Fux** wrote about the abolition of un-equal half tones.
- **1727 Meckenheuser** calculated and described equal temperament.
- **1739 Mizler** promoted equal temperament.
- **1744 Sorge** calculated equal temperament.
- **1746 Tans’ur** tuned all the thirds as wide as possible.
- **1756 Fritz** describes his method which he had used already applied for 40 years to tune equal temperament by ear.
- **1753 Corrette** mentioned several possibilities to tune, including equal temperament.
- **1758 Adlung** mentioned that Werckmeister introduced the equal temperament in Germany, a temperament supported by famous writers such as Mattheson, Marpurg, Petri and many writers in the 19th century.
- **1760 Kirnberger** published his: Construction der gleichschwebende Temperatur (Construction of equal temperament) and his three alternative un-equal temperaments.
- **1776 Marpurg** quoted Barthold Fritz’s method in his chapter: How to tune equal temperament.
- **1782 Petri** quoted Barthold Fritz’s method to tune equal temperament in connexion with the ‘unsuspected modulations’ used by J.S. Bach.\(^{156}\)

\(^{155}\) Anleitung zur praktischen Musik page 100 and 373.

\(^{156}\) See the details in the last quote from the Bach Documents
Part Two

In Europe from the early 17th century onwards to the end of the late 18th century several temperaments were known. Werckmeister and Neidhardt were frequently mentioned in the important 18th century books about music, but almost always, Werckmeister and Neidhardt were quoted in connection with equal temperament. As gleaned from Part 1 various theoreticians calculated and invented new tunings, but not every calculated temperament was used by musicians. Invariably, musicians tuned by ear. We have described how equal temperament was introduced in Germany and that during Bach’s lifetime meantone, unequal and equal temperament were in use. We have also shown that different theoreticians, composers and musicians had different opinions regarding temperaments. Now we shall compile a sequence of all the existing quotations from the 18th century on J.S. Bach and temperament to have a look at how Bach tuned according to his contemporaries.

The Bach Documents is a supplement to the New Bach Edition (Bach Dokumente in German). Prior to the most recent edition of the Bach Dokumente being republished in 2007, the German musicologist Christopf Wolff went to the former East Germany in search for new documents on Bach. Everything written in the 18th century on J.S. Bach and which survived is compiled in the Bach Dokumente, except for one 18th century quotation which connects J.S. Bach with equal temperament and which was omitted in the Bach Dokumente. We have added this quotation at the end of Dokument 868.

To understand the Bach Documents properly it is necessary to understand the difference between equal temperament with equal beating fifths and equal temperament with inproportion beating fifths which we described in Part 1.

Bach Dokumente Book I
Documents written by Johann Sebastian Bach.

Document 85, 1716
Bach and the organist and composer Johann Kuhnau tried out the new Cuncius organ and the report is signed by Kuhnau and J.S. Bach.

‘Doch hat Herr Cuncius, gleich wie er noch hin und wieder das Werck, welches wir in allen 3 Clavieren noch ziemlich unrein befunden, reiner zu stimmen, und nach der von ihm uns einmalh gezeigten noch passablen guten Temperatur einzurichten versprochen.’

‘All three manuals are rather out of tune, but Mr Cuncius promised us now to tune the organ better, accordingly to the still acceptable good tuning which he showed us once.’

‘Still acceptable good tuning’ implies that there is a better tuning. According to Werckmeister and Adlung acceptable good temperament means almost equal temperament, but their first choice was equal temperament.

157 Information provided by Reinhard Goebel.
158 Bach-Dokumente, supplement zu Johann Sebastian Bach Neue Ausgabe Sämtlicher Werke, Bärenreiter 2007, Volume V, Sachbegriffe: Temperatur page 341
159 See the letter published by Mattheson in his Critica Musica (1725) page 229-239 about Kuhnau and Neidhardt’s equal temperament.
We shall see later what Bach’s preferred temperament was.

**Document 87, 1717**
This document is about intonation of organ pipes, and some other problems, but does not give information about tuning.

**Document 152, 1722**
Handwritten Bach *Das Wohltemperirte Clavier*

What *Wohltemperir*rt means to Werckmeister at the end of his life, can be found in his *Musikalische Paradoxal-Discurso* which was published in 1707:

‘Wir wissen/ wenn die Temperatur also eingerichtet wird/das alle Quinten 1/12 Comma schweben und ein accurates Ohr dieselbe auch zum Stande zu bringen/und zustimmen weiß/ so dann gewiss eine wohl temperirte Harmonia, durch den ganzen Circul und durch alle Claves sich finden wird. Welches dann ein Vorbild seyn kan/ wie alle fromme/und wohl temperirte Menschen/ mit Gott in stets währender gleicher/ und ewiger Harmonia leben/und jubiliren werden’.

‘We know, if the temperament is set up so that all fifths beat by 1/12 of a comma and an accurate ear is able to see to this and to tune like that, then certainly a well-tempered harmony will result throughout the cycle (of fifths) and in all keys. This can be taken as an image of all pious and well-tempered people, who will live and rejoice with God in a constant, equal and eternal harmony.’

And C.P.E Bach wrote about *Das Clavier*:
‘Auf dem Claviere spielet man aus allen vier und zwanzig Ton=Arten gleich rein’.

‘On the Clavier one plays in all twenty four keys equally in tune’.

**Bach Dokumente Book II**
Fremdliche und gedruckte Dokumente (Printed documents by others)

**Document 63**
About the employment of Bach as organ player in Halle in 1713:

‘Ferner wird Er (J.S.Bach) sich befleißigen...daß die eingepfarrte Gemeinde die Orgel zum Fundamente einer guten Harmonie und gleichstimmig Thones sezen, darinn andächtig singen, und dem Allerhögsten dancken und loben mögen’

‘In addition he (J.S. Bach) will exert himself to use the organ to bring the church community in a fundamentally good harmony and an equally tempered mood.’

As part of Bach’s duties he also needed to take care that:
‘Das Orgel in guten Stande, auch rein gestimmet und ohne dissonantz erhalten’.

‘The organ is well maintained, justly tuned and remains without dissonance.’

160 Mus. Par. Dis. Page 110
Dokument 514
Voigt: Schwierigkeit Bachser Klavierwerke, 1742.

‘Vor einiger Zeit habe ich die Ehre gehabt, Monseur Krebsen\textsuperscript{161}, den neuen Organisten in Zwickau, einen sehr starken Clavier- und Orgel-spieler, zu sprechen und zu hören. Ich muß gestehen, daß es etwas wichtiges sey, was dieser Mensch, als ein Organiste, vor andern thut, und ist er eine Bachische Creatur, und bestehet die Rendresse in schweren und geschwinden Ausübungen durch alle 3 Genera unsers disponirten Claviers, oder temperirten Orgel-Wercks’.

‘Some time ago I had the pleasure of speaking to and hearing Mister Krebs, the new organist at Zwickau, a very good keyboard and organ player. I must say, it is very important what this man, as an organist, does better than others. He is creature like Bach: ‘And he stands his ground in difficult and fast executions through all 3 genera on our well-made claviers or tempered organs.’

Dokument 575
Sorge: Mängel der Silbermann-Temperatur\textsuperscript{162}, 1748

‘Es ist genug , daß die Quinte gs: ds unleidlich über sich schwebet, welches keineswegen zu leugnen; und diese bezeuget genugsam, daß die übrigen 11. wo nicht alle, jedoch die meisten zu viel abwärts schweben, welches so dann verursacht, daß 4. grosse Tertzen allzu rauh, scharff und barbarisch, und 3 kleine allzu weich, faul und träge werden... Daß diese alles die lautere Wahrheit sey, ruffe ich alle unpartheyische und der Sache erfahrne Musicos, sonderlich den Welt-berühmten Herrn Bach in Leipzig zu zeugen’.

‘In denen 4. schlimmen Triadibus aber ist ein rauhes, wildes, oder, wie Herr Capellmeister Bach in Leipzig redet, ein barbarisches Wesen enthalten, welches einem guten Gehör unerträglich fällt.’

‘It is enough (to know) that the fifth g#-d# intolerable wide is, which cannot be denied; and this proves sufficiently that the other 11, if not all, than the most of them, are too narrow, resulting in four major thirds being very harsh, fierce and barbaric, and three minor (thirds) being far too soft, lazy and sluggish.

I call on all unbiased musicians experienced in the matter and especially the world renowned Mr Bach from Leipzig to confirm that this is the plain truth.’

The four bad triads have a harsh, wild or - as Mr Capellmeister Bach from Leipzig says – a barbaric character, which is unbearable to a good ear.’

In other words, Bach said, accordingly to Sorge, that the Silbermann-Temperament is insupportable for the ear.’

\textsuperscript{161} http://en.wikipedia.org/wiki/Johann_Ludwig_Krebs
\textsuperscript{162} Shortcoming in the Silbermann’s temperament;
`Die Clavicymbale wußte er, in der Stimmung, so rein und richtig zu temperiren, daß alle Tonarten schön und gefällig klangen. Er wußte, von keinen Tonarte, die man, wegen unreiner Stimmung, hätte vermeiden müssen.

‘He (Bach) knew how to temper and tune a harpsichord in such a just and right way, that all keys sounded beautiful and pleasing. He did not know any keys that one would have to avoid because of unequal tuning.’

In the 18th century often the word ‘rein’ (pure) is used in combination with ‘temperament’. Pure doesn’t mean that intervals are pure; temperament means that intervals are a bit out of tune, only octaves are pure. Several writers (Werckmeister, Sorge and Petri) used ‘reinen Temperatur’ for equal temperament.

‘Das temperirte Clavier in 2 Theilen…
Jeder Theil enthält 24 Stücke, aus jeder Tonart ein Präludium und eine Fuge, und hat daher den Namen, weil ein Clavier muß temperirt, oder in allen Tonarten brauchbar seyn, auf welches man solche spielen soll.’

‘The tempered keyboard in 2 parts…Each part contains 24 pieces, a prelude and a fugue in each key. The title implies that a keyboard must be tempered, or usable in all keys, if one wants to play these (pieces) on it.’

‘Wer nun diese Verschiedenheiten der Terzien und übrigen Verhältniße mit dem Gehöre unterscheidet, (und sie sind zu unterscheiden) wird ohne boshafte Absichte ein Stück nicht in einen andern Ton transponiren. Denn nicht ein einziges Stück von seligen Bach, Graun, Hendel, Herrn Capellmeister Bach in Hamburg und andern großen Componisten, kann in einen andern Ton versetzt werden, ohne es zu verunstalten und unpracktikabel zu machen.’

‘Whoever is able to discern the difference of thirds and other intervals with his ears (and they can be differentiated) will not transpose a piece into another key without bad faith. Because not even one piece by the late Bach, Graun, Hendel (sic) Capellmeister Bach in Hamburg and other great composers can be transposed into another key without deforming it and making it impracticable.’
Document 758
Johann Ernst Altenburg (1734-1801): Unterricht bei Johann Christoph Altnicol, nach 1769
(Lessons with Johann Christoph Altnicol, after 1769)

‘Herrn Altnicol (der ein würdiger Schwieger Sohn, des weltberühnten Johann Sebastian
Bachs war,) mit guten Erfolge geschahe, bey welchem er zugleich richtige Begriffe u.
Kenntniße vom Orgelbau, u. reinen Stimmung (Temperatur) profitirte.’

‘Mr Altnicol (who was the honourable son-in-law of the world renewed Johann Sebastian
Bach) to great success, where he also learned about the terminology and building of organs,
and about just tuning (temperament).’

Dokument 767
Marpurgs Kritik über die Kirnbergischen Temperatur. (Critics on Kirnberger’s temperament)
doesn’t give information about Bach’s temperament.

Dokument 772
Forkel: Bedeutungslosigkeit der Mathematik für Bachs Temperierungsmethode, 1772
(Mathematics is not important for Bach’s way of tuning, 1772)

‘Selbst der in der Mathematik so gelehrte Johann Sebastian Bach habe in diesen fragen nach
der Natur, nicht nach der Regel gerichtet, und die ganze Mathematisiererei habe noch nicht
einmal den Erfolg gehabt, die Durchführung einer einwandfreien temperatur zu gewähleisten.’

‘Even Johann Sebastian Bach, well versed in mathematics, did not go by the rule book in
these matters but accordingly to nature. And all mathematics did not even succeed in
securing an acceptable temperament.’

Here we find a similarity with the difference between Pythagoras who calculated tunings and
Aristoxenus who tuned by ear.

Dokument 801
C.P.E Bach: Biographische Mitteilungen über Johann Sebastian Bach, (Biographical remarks
on J.S. Bach) Hamburg, Ende 1774.

‘Das reine stimmen seiner Instrumente so wohl, als das ganze Orchestres war sein
vornehmstes Augenmerck. Niemand konnte Ihm seine Instrumente zu Dancke stimmen u.
bekielen’.

‘He turned his complete attention to the just tuning of his instruments as well as the
orchestra. No one could expect thanks for tuning or putting new plectra on his instruments.’

Dokument 815
Marpurg: Kritik an Kirnbergers Temperaturvorschlägen, 1776 (Criticism on Kirnberg’s
temperament suggestions, 1776)

‘Man komme mir hier mit keiner Autorität aus den vorigen Jahrhunderten163 wo man drey
Tonarten häßlich machte, um eine einzige recht schöne zu erhalten…. Der Hr. Kirnberger hat

163 17th century
mir und andern mehrmals erzählt, wie der berühmte Joh. Seb. Bach ihm, während der Zeit
seines von derselben genoßn Unricht, die Stimmung seines Claviers übertragen, und
die dieser Meister ausdrücklich von ihm verlanget, alle großen Tertsen etwas scharf, d.i. wo
sie alle über sich schweben sollen, kann unmöglich eine reine große Terts statt finden…. Der
Hr. Capellmeister Joh. Seb. Bach, welcher nicht ein durch einen bösen Calcul verdorbnes
Ohr hatte, mußte also empfunden haben, daß eine um 81:80 erhöhte Tertz ein abscheuliches
Interval ist. Warum hatte derselbe wohl seine aus allen 24 Tönen gesetzte Präludien und
Fugen die Kunst der Temperatur betitelt?

‘Don’t refer me to any authority from the previous centuries, when three keys were made ugly
in order to get one quite beautiful… Mr Kirnberger told me and others several times, how the
well-known Joh. Seb. Bach entrusted him with the tuning of his harpsichord while he
[Kirnberger] had lessons with him [Bach]. And how this master expressly required him to
tune all major thirds a little sharp, i.e. since they all beat, it is impossible to tune a just major
third; and since no just major third is tuned, no major third tempered 81:80 is possible. Mr
Capellmeister Joh. Seb. Bach, whose hearing wasn’t corrupted by bad calculations, therefore
must have found a third tempered 81:80 abominable. Why else would he have called his
Preludes and Fugues in all 24 keys the art of temperament?’

Dokument 868
Petri: Bachs Klaviertechnik- Notwendigkeit temperierter Stimmung Bautzen, 1782 (Need of
a temperament).{

Petri was a student and friend of Wilhelm Friedrich Bach and is the last 18th century writer
who wrote about Bach and temperament.

‘Indeß erfand Johann Sebastian Bach zu Leipzig die rechte Art, das Klavier zu spielen, und
machete seine Partien durch den Kupferstuch bekannt. Nun spielte Deutschland und
Frankreich bachisch, und in England zeigte Händel ebenfalls, wozu das Klavier zu brauchen
sey.
Hatten die vorhin genannten Komponisten eine reine Temperatur noch nicht nothwendig
genug gemacht, so machte sie doch der Leipziger Bach mit seinen tiefssinnigen und
unerwarteten Gängen in bisher ganz ungebrauchter Töne nun ganz unentbehrlich. Denn nun
sahe man, daß die besten Klavierinstrumente wegen ihrer unreinen Stimmung unbrauchbar
waren, selbst die schönen Silbermannischen orgeln waren falsch gestimmt. [S. 100].

Bachs würdige Söhne machten das Klavier durchgängig bekannt und beliebt, besonders Karl
Philipp Immanuel, welcher erst in Berlin war, jetzt aber in Hamburg ist; sodann auch der
Mailändische, der jetzt in London als Königlicher Kapellmeister mit Ruhme steht.’ [S. 101].

‘Meanwhile Johann Sebastian Bach in Leipzig invented the true art of playing the keyboard
and distributed his Partitas by printing them. Now Germany and France played like Bach,
and in England Handel also showed how to use a keyboard instrument.
Though the composers mentioned previously did not really need a ‘reine Temperatur’, for
Bach in Leipzig with his profound and unexpected modulations into hitherto unused keys it
became truly indispensable. Now it became obvious that the best keyboard instruments were

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164 Petri’s quote as it is compiled in the Bach Dokumente. The quote is from Petri’s, Anleitung zur praktischen
unusable because of their un-equal temperament, even the beautiful organs by Silbermann were tuned wrongly. (Page 100)

‘Bach's worthy sons made the keyboard continuously known and loved, especially Karl Philipp Emmanuel, who first lived in Berlin, now in Hamburg; but also the Milanese, who is now in London as the Kings famous Kapellmeister. (Page 101)

**Quote omitted in the Bach Dokumente.**

The Bach Documents were first compiled in the D.D.R. (former East Germany) and Petri’s quote in the Bach Documents mentions that Bach’s modulations made a ‘pure Tempertur’ (pure, clean temperament) necessary. The Bach Dokumente gives a quote from page 100 and continues with a quote from page 101. For some reason the Bach Dokumente omitted an important part of the text on page 100
d, namely Petri's description of what 'pure Tempertur' is:

‘Heinichen und Sorge nebst andern wollten diesem Uebel auf Pythagorische Art abhelfen; allein demonstrationen und Berechnungen halfen nichts, bis endlich die Instrumentmacher auf Aristoxenische Art die reine temperatur durch ein gutes Gehör, und durch Herablassung der zu scharf gestimmten Quinten erfanden. Ja, Barthold Fritz, Instrumentmacher in Braunschweig, machte diese Methode zu stimmen durch ein paar gedruckten Bogen öffentlich bekannt.’

‘Heinichen and Sorge wanted to solve the problem in Pythagorean manner, but all demonstrations and calculations came to nothing, until the instrument makers in Aristoxanean manner invented the right temperament trusting a good ear and tuning the sharp fifths narrow. Yes, Barthold Fritz, even made this method of tuning known to the public by publishing a few papers.’

Petri explains on page 373 how Fritz’ temperament is tuned: **12 fifths equally tempered, by ear.**

And so we are now back with Barthold Fritz whom we met in the first half of the article: according to Petri, Bach did not use equal temperament with in proportion beating fifths, such as Heinichen and Sorge calculated with help of a monochord, but equal temperament with equal beating fifths tuned by ear.

**Synopsis Part Two.**

In 1716 Kuhnau and Bach criticized the new Cuncius organ and found the ‘still acceptable good tuning’ not good enough.

In 1748 Sorge criticized the Silbermann tuning, referring to J.S. Bach.

1754 Obituary of J.S. Bach: Bach tuned so that all keys sounded good.

Forkel wrote in 1772: Bach tuned by ear and did not calculate temperament.

Marpurg writes, accordingly to Kirnberger, that Bach insisted to tune all thirds wide.

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165 During the communist regime in the D.D.R. it was not unusual not to publish certain information which was considered unsuitable for a wider audience.

166 Petri *Anleitung zur praktischen Musik*, first print 1767, second print 1782 page 100
In 1782 Petri writes that because of J.S. Bach’s unexpected modulations into hitherto unused keys a ‘reine Temperatur’ became truly indispensable and this ‘reine Temperatur’ is ‘equal temperament’ tuned by ear as described by Barthold Fritz.
Appendix I: Marpurg, *Versuch*, page 231

...gatt, wenn mit Autoritäten gestritten werden soll. Der Herr Kirnberger selbst hat mir und anderen mehrmals erzählt, wie der berühmte J. S. Bach ihm, während der Zeit seiner von ihm selbst genossenen musikalischen Unterricht, die Stimmung seines Claviers übertragen, und wie dieser Meister ausdrücklich von ihm verlangt, alle große Terzen scharf zu machen. In einer Temperatur, wo alle große Terzen etwas scharf, d. i. wo sie alle über sich schweben sollen, kann unmöglich eine reine große Terz stattfinden, und sobald keine reine große Terz statt findet, so ist auch keine um 81:80 erhöhte große Terz möglich. Der Herr Capellmeister J. S. Bach, welcher nicht ein durch einen bösen Calcul verdorbnes Ohr hatte, musste also empfinden haben, daß eine um 81:80 erhöhte große Terz ein abscheuliches Intervall ist. Warum hatte derjenige wohl seine aus allen 24 Tönen gesetzte Präludien und Fugen die Kunst der Temperatur beteiligt?
Appendix II: Mattheson, Ehrenpf. page 231

Lernet. Die Querflöte hat er von sich selbst ganz allein getrieben, so daß er, nach der Zeit, als ein Student, sich öfters darauf hören lassen. In der Composition hat er sich durch Lesung guter Bücher; Anhörung guter Musik; Durchsicht vieler Partituren von guten Meistern, und auch durch den Umgang mit dem Capellmeister Bach festgelegt. Besonders gesteht er, aus den Schriften des berühmten Capellmeisters Mattheson viel Gutes in seinem Musik verwenden zu haben. (Sehr wohl!)

Die bisherigen musikalischen Schriften sind folgende: 1.) Dissertatio, quod musica scientia sit & pars oruditionis philosophiae. Edit. secunda, 1736. 2.) De usu & praestantia Philosophiae in Theologia, Jurisprudentia, Medicina. 3.) Musikalische Bibliothek, oder gründliche Nachricht, nebst unparteischem Urtheil von musikalischen Schriften und Büchern. 4.) Anfanggründe des Generalbaßes, nach

Es wäre zu wünschen, daß er sich mögte bewegen lassen, der musikalischen Welt mit dem Raume der äußersten Stüde, seiner eigenen Composition bald eine Luft zu machen.

Diefer hat ihm gewiß und wahrhaftig auch so wenig die vermeinten mathematischen Composition-Gründe beigebracht, als der nachstgenannte. Dafür bin ich Bürge.


er sie im Kopfe hatte. Die Clavicymbale mußte er, in
der Societ. der mus. Wissenschaften. 173

in der Stimmung, so rein und richtig zu temperieren, daß alle Tonarten schön und gefällig klangen. Er mußte, von seinen Tonarten, die man, wegen unreiner Stimmung, hätte vermeiden müssen. Andere Vorzüge, die ihm eigen waren, zu geschweigen.
Auch stimmte er so wohl den Flügel als sein Clavichord selbst, und war so geübt in dieser Arbeit, daß sie ihm nie mehr als eine Viertelsekunde kostete. Dann waren aber auch, wenn er fantasirte, alle 24 Tonarten sein; er machte mit ihnen was er wollte. Er verband die entferntesten so leicht und so natürlich mit einander, wie die nächsten; man glaubte, er habe nur im inneren Kreise einer einzigen Tonart modulirt. Von Härten in der Modulation wußte er nichts; seine Chromatik sogar war in den Übergängen so sanft und fließend, als wenn er bloß im diatonischen Klanggeschlecht geblieben wäre. Seine nun schon geschnürt sogenannte chromatische Fantasie kann beweisen, was ich hier sage. Alle seine freyen Fantasien sollen von ähnlicher Art, häufig aber noch weit freier, glänzender und ausdrucksvoller gewesen sein.

Appendix V: Mersenne’s monochord in equal temperament
Appendix VI: Galle’s early 17th century calculation of equal temperament as printed in Mersenne’s *Nouv. Obs.* page 21

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